



## Low frequency absorption by velocity control through acoustic resistance

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### 1. About the problem

All premises used for sound measurement, recording, processing and diffusion, such as recording or post production studios, concert halls, sound laboratories, home theaters and listening rooms need to be acoustically treated to obtain the adequate reverberation and echo that is required by for their use.

In a standard sized room, the natural standing resonances occur in general at low frequency and therefore represent a serious problem to be dealt with.

### 2. About passives solutions

Several passives attempts to solve this problem have been made but they all have many limitations, in particular:

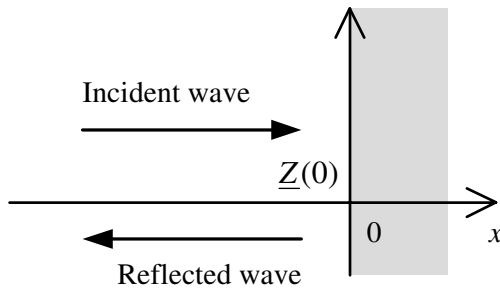
- The important size of the passive porous absorbers (thicknesses increases with the wave length: as an example, a minimum of 1 m of material is necessary to suitably absorb frequencies of 100 Hz);
- The narrow bandwidth of the resonator systems [1], e.g. Helmholtz, membrane resonator, etc.

### 3. One-dimensional wave reflection and impedance

In a one-dimensional wave the pressure and particle velocity wave phasors are given by:

$$\begin{aligned} \underline{p} &= \underline{p}_{0i} e^{-jkx} + \underline{p}_{0r} e^{+jkx} \\ \underline{v} &= \underbrace{\underline{v}_{0i} e^{-jkx}}_{\text{incident}} + \underbrace{\underline{v}_{0r} e^{+jkx}}_{\text{reflected}} \end{aligned} \quad \text{with} \quad \begin{aligned} \underline{p}_{0i} &= \underline{v}_{0i} \cdot Z_c \\ \underline{p}_{0r} &= -\underline{v}_{0r} \cdot Z_c \end{aligned}$$

$k = \frac{2\pi}{\lambda}$  is the wave number and  $\lambda$  is the wavelength.



We assume the wall to be normal to the direction in which the incident wave is travelling.

The surface wall impedance is given by:  $\underline{Z}(0) = \frac{p(x=0)}{v(x=0)} = Z_c \frac{\underline{p}_{0i} + \underline{p}_{0r}}{\underline{p}_{0i} - \underline{p}_{0r}}$

So the **pressure reflection coefficient** is:  $\underline{r}(0) = \frac{\underline{p}_{0r}}{\underline{p}_{0i}} = \frac{\underline{Z}(0) - Z_c}{\underline{Z}(0) + Z_c}$

With  $\frac{\underline{Z}(0)}{Z_c} = \underline{\zeta}$ , we obtain  $\underline{r}(0) = \frac{\underline{\zeta} - 1}{\underline{\zeta} + 1}$ .

In the same way, we have:  $\underline{v}_{0r} = -\underline{r} \cdot \underline{v}_{0i}$

### 3.1. Particular case: rigid wall

On a rigid wall, with  $v(x=0) = 0$ , we have:  $\underline{Z}(0) = \frac{p(x=0)}{v(x=0)} = \infty$  and  $\underline{r} = 1$ .

$$\underline{p} = \underline{p}_{0i} (e^{-jkx} + e^{+jkx}) = 2 \underline{p}_{0i} \cos(kx) \quad \text{and} \quad p(x, t) = 2 \hat{p}_{0i} \cos(kx) \cos(\omega t)$$

$$\underline{v} = \underline{v}_{0i} (e^{-jkx} - e^{+jkx}) = 2j \underline{v}_{0i} \sin(kx) \quad \text{and} \quad v(x, t) = 2 \hat{v}_{0i} \sin(kx) \sin(\omega t)$$

RMS acoustic pressure and velocity varies between zero and twice the free field values.

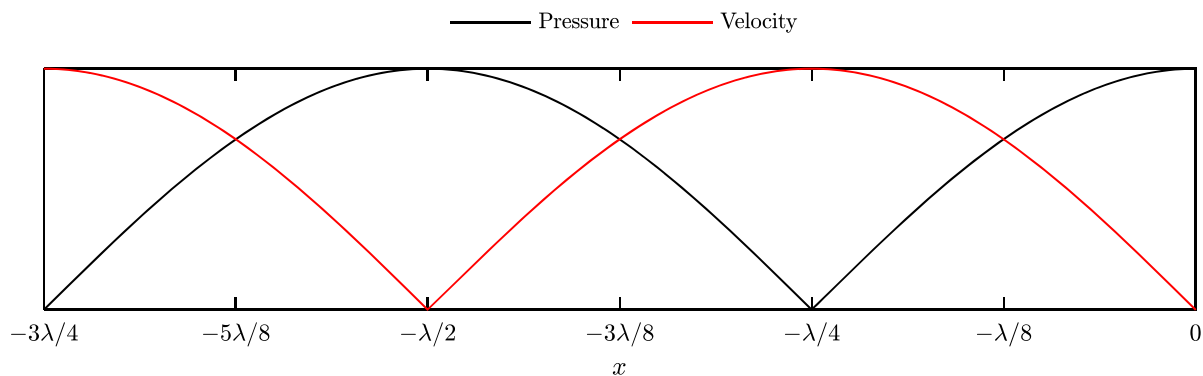


Figure 1. Relative pressure and velocity in front of a rigid wall.

#### 4. One-dimensional wave intensity and absorption coefficient

The *acoustic intensity*, defined as the time average of power transmission through a unit area normal to the direction of propagation of the wave, is given by:  $I = \overline{p \cdot v} = \underline{p} \cdot \underline{v}^*$  (W/m<sup>2</sup>)

So:  $I_{reflected} = -\underline{r} \cdot \underline{r}^* \cdot \underline{p}_{0i} \cdot \underline{v}_{0i}^* = -|r|^2 I_{incident}$

The *intensity absorption coefficient* is defined as:  $\alpha = \frac{I_{incident} - I_{reflected}}{I_{incident}} = \frac{I_{absorbed}}{I_{incident}} = 1 - |r|^2$

And with the wall surface impedance ratio  $\underline{\zeta}$ :  $\alpha = 1 - |r|^2 = \frac{4 \operatorname{Re}(\underline{\zeta})}{|\underline{\zeta}|^2 + 2 \operatorname{Re}(\underline{\zeta}) + 1}$

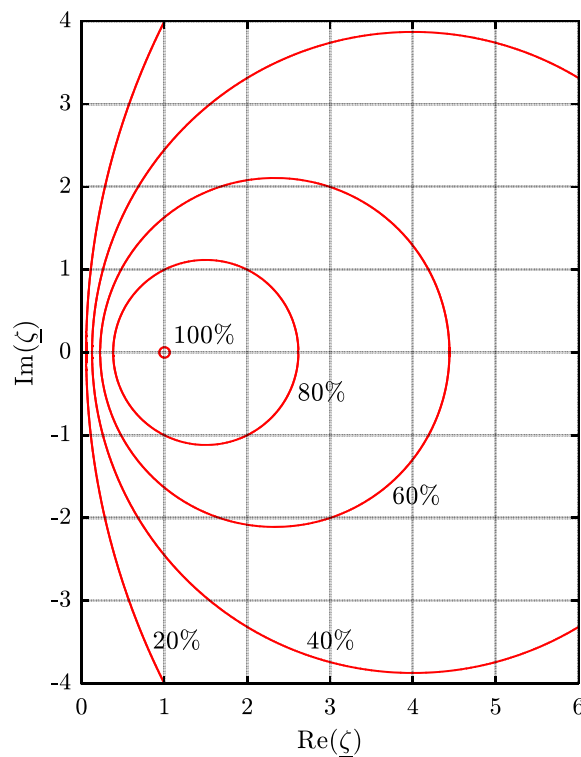


Figure 2. Intensity absorption coefficient  $\alpha$  as a function of the impedance ratio  $\underline{\zeta}$  in complex plane [1].

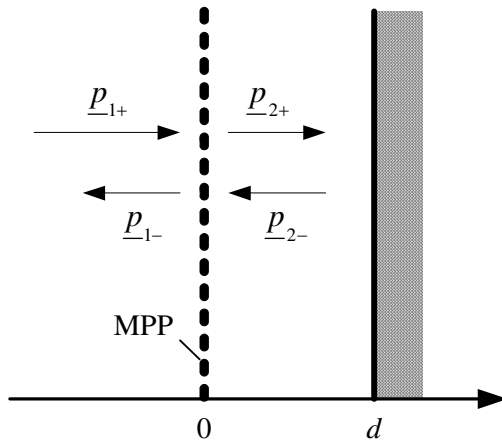
#### 5. Sound absorption of a thin porous layer in front of a rigid wall

We consider a thin porous layer, e.g. a stretched fabric or a micro perforated panel (MPP), at distance  $d$  in front of a rigid wall. We assume the wall to be normal to the direction in which the incident wave is travelling.

Any pressure difference between the two sides ( $p_1 - p_2$ ) of the layer forces an air stream through the pores with an air velocity  $v$ .

We assume a resistive layer and Darcy's law:  $\underline{p}_1 - \underline{p}_2 = R \underline{v}$ , where  $R$  is the *flow resistance* (Pa·s/m) of the layer.

The layer is located at  $x = 0$  and the wall at  $x = d$ .



With the pressures wave:

- In front of the MPP (medium 1):

$$\underline{p}_1 = \underline{a}_1 e^{-jkx} + \underline{b}_1 e^{+jkx} \quad \text{for } x < 0$$

- Behind the MPP (medium 2):

$$\underline{p}_2 = \underline{a}_2 e^{-jkx} + \underline{b}_2 e^{+jkx} \quad \text{for } x > 0$$

- $\underline{a}_1$  and  $\underline{a}_2$  are the incident pressure wave phasors for  $x = 0$  ( $x \rightarrow 0$ ),
- $\underline{b}_1$  and  $\underline{b}_2$  are the reflected pressure wave phasors for  $x = 0$  ( $x \rightarrow 0$ ).

The medium 1 and 2 present the characteristic impedance  $Z_c$  and the progressive and retrograde waves are given by:

$$\begin{aligned} \underline{p}_{1+} &= Z_c \underline{v}_{1+} & \underline{p}_{1-} &= -Z_c \underline{v}_{1-} \\ \underline{p}_{2+} &= Z_c \underline{v}_{2+} & \underline{p}_{2-} &= -Z_c \underline{v}_{2-} \end{aligned} \quad \text{and}$$

The velocities are:

- $\underline{v}_1 = Z_c^{-1} (\underline{a}_1 e^{-jkx} - \underline{b}_1 e^{+jkx}) \quad \text{for } x < 0,$
- $\underline{v}_2 = Z_c^{-1} (\underline{a}_2 e^{-jkx} - \underline{b}_2 e^{+jkx}) \quad \text{for } x > 0.$

**On the interface** (MPP), we have:

- Flow continuity:  $\underline{v}_1(0-) = \underline{v}_2(0+)$
- A pressure loss:  $\underline{p}_1(0-) - \underline{p}_2(0+) = R \underline{v}(0)$

**On the wall:**

- $\underline{v}_2(d) = 0 = Z_c^{-1} (\underline{a}_2 e^{-jkd} - \underline{b}_2 e^{+jkd})$

There is 3 relations between  $\underline{a}_1, \underline{b}_1, \underline{a}_2, \underline{b}_2$ :

$$\begin{pmatrix} 1 & 1 & -1 \\ \rho + 1 & -1 & -1 \\ 0 & e^{-jkd} & -e^{jkd} \end{pmatrix} \begin{pmatrix} \underline{b}_1 \\ \underline{a}_2 \\ \underline{b}_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \rho - 1 \\ 0 \end{pmatrix} \underline{a}_1 \quad \text{with} \quad \rho = R / Z_c$$

So: 
$$\underline{b}_1 = \frac{\cos(kd) + j(\rho - 1)\sin(kd)}{\cos(kd) + j(\rho + 1)\sin(kd)} \underline{a}_1,$$

$$\underline{a}_2 = \frac{\exp(jkd)}{\cos(kd) + j(\rho + 1)\sin(kd)} \underline{a}_1 \quad \text{and} \quad \underline{b}_2 = \frac{\exp(-jkd)}{\cos(kd) + j(\rho + 1)\sin(kd)} \underline{a}_1$$

The MPP surface impedance is given by: 
$$\underline{Z}(0) = Z_c \left( \frac{\underline{a}_1 + \underline{b}_1}{\underline{a}_1 - \underline{b}_1} \right) = R - jZ_c \cot(kd)$$

The absorption coefficient is: 
$$\alpha = 1 - |\underline{r}|^2 = \frac{4\rho}{(\rho + 1)^2 + \cot^2(kd)}$$

If  $\rho = 1$  and  $\cot(kd) = 0$ , *the absorption is maximal*. This maximum appears for  $d = (2n + 1)\frac{\lambda}{4}$ .

**Example** with  $c = 330$  m/s,  $\rho = 1$  and  $d = 25$  cm:

We obtain a first maximum for  $f_0 = 330$  Hz, then  $f_1 = 3 \cdot 330 = 990$  Hz, etc.

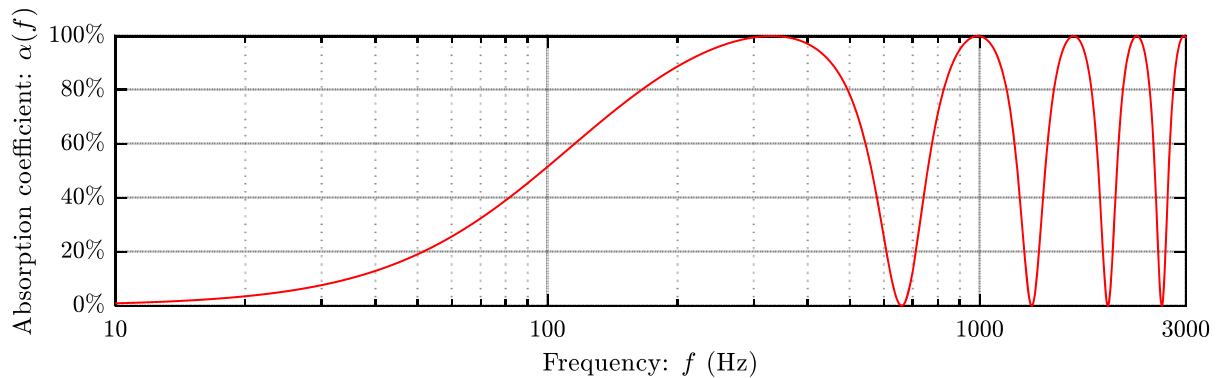


Figure 3. Absorption coefficient vs. frequency.

The RMS rear pressure is: 
$$|\underline{p}_2(x=0)| = |\underline{a}_2 + \underline{b}_2|$$

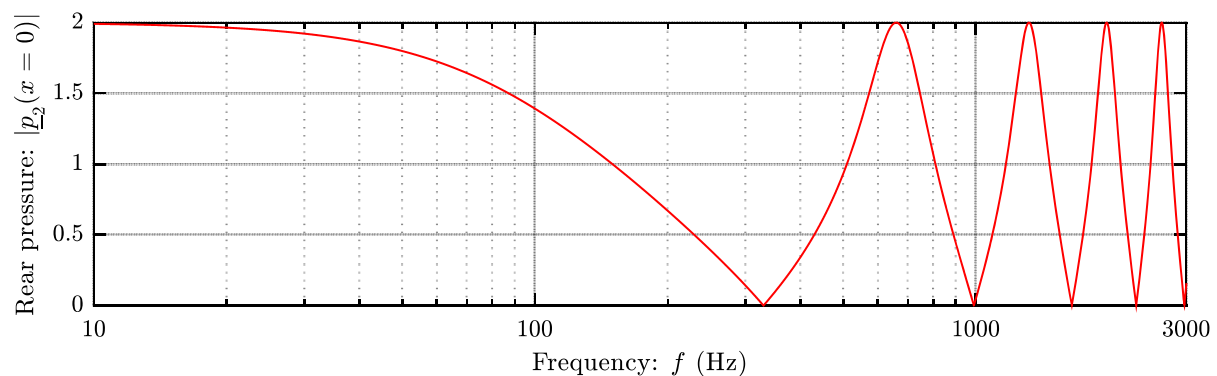


Figure 4. RMS rear pressure vs. frequency.

We can see that the rear pressure is null for the same frequencies. In these cases, the MPP surface impedance is equal to the MPP flow resistance  $R$ .

**Example** of RMS pressure and velocity at  $f = 200$  Hz and  $f = 330$  Hz.

The MPP is located at  $x = 0$ .

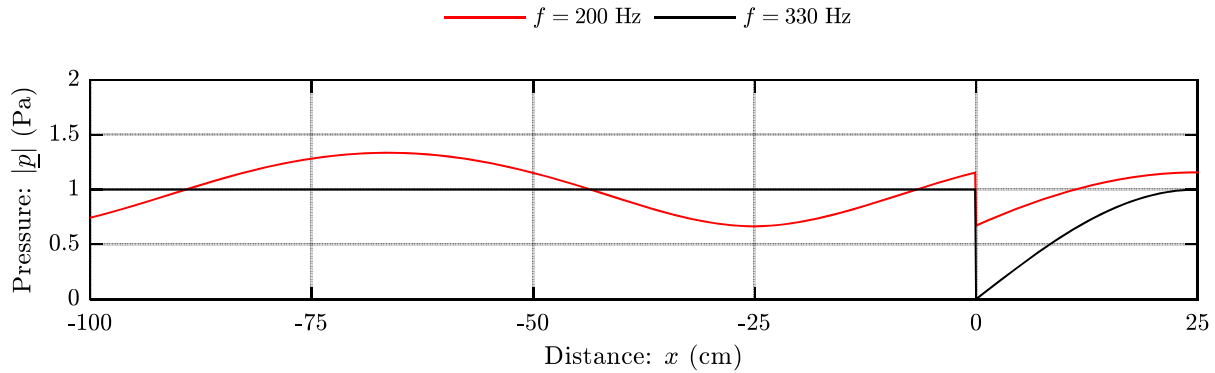


Figure 5. RMS pressure vs. distance. Note the discontinuity at  $x = 0$ .

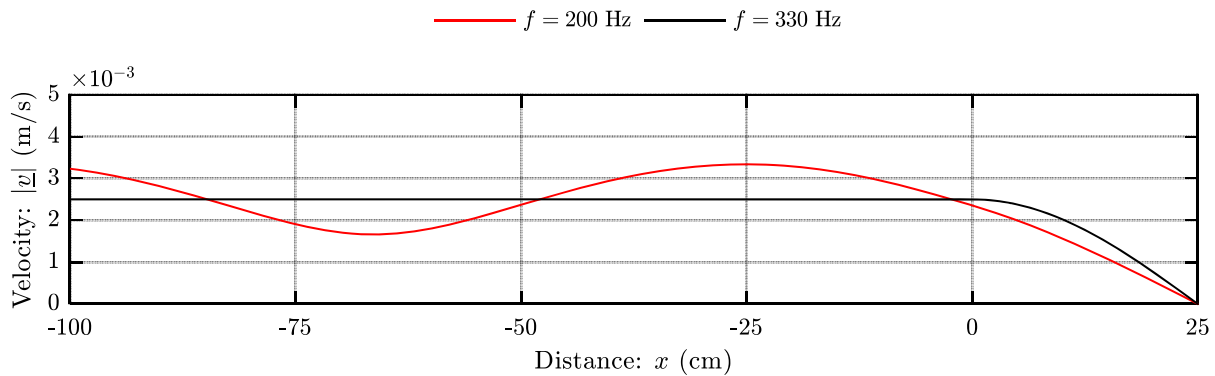


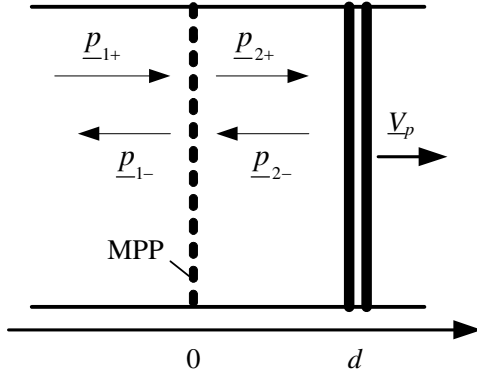
Figure 6. RMS velocity vs. distance. Note the continuity at  $x = 0$ .

We can see, for  $f = 330$  Hz ( $\lambda / 4 = 25$  cm):

- A maximal pressure loss across the resistance;
- No reflected wave, given by a constant RMS incident pressure value (1 Pa);
- Independently of the frequency, the continuity of the velocity across the resistance.

## 6. Sound absorption of a thin porous layer in front of a moving wall

We consider the same system in a tube, with the thin porous layer at distance  $d$  in front of a moving piston of given velocity  $\underline{V}_p$ .



With the pressures wave:

- In front of the MPP (medium 1):

$$\underline{p}_1 = \underline{a}_1 e^{-jkx} + \underline{b}_1 e^{+jkx} \quad \text{for } x < 0$$

- Behind the MPP (medium 2):

$$\underline{p}_2 = \underline{a}_2 e^{-jkx} + \underline{b}_2 e^{+jkx} \quad \text{for } x > 0$$

With the same notations and the new condition of continuity on the wall:

$$\underline{v}_2(d) = \underline{V}_p = Z_c^{-1} (\underline{a}_2 e^{-jkd} - \underline{b}_2 e^{+jkd})$$

The new equations are:

$$\begin{pmatrix} 1 & 1 & -1 \\ \rho + 1 & -1 & -1 \\ 0 & e^{-jkd} & -e^{jkd} \end{pmatrix} \begin{pmatrix} \underline{b}_1 \\ \underline{a}_2 \\ \underline{b}_2 \end{pmatrix} = \begin{pmatrix} \underline{a}_1 \\ (\rho - 1) \underline{a}_1 \\ Z_c \underline{V}_p \end{pmatrix}$$

So:  $\underline{b}_1 = \frac{(\cos(kd) + j(\rho - 1)\sin(kd)) \underline{a}_1 - Z_c \underline{V}_p}{\cos(kd) + j(\rho + 1)\sin(kd)}$ ,

$$\underline{a}_2 = \frac{\exp(jkd) \underline{a}_1 - 0.5 \cdot Z_c \rho \underline{V}_p}{\cos(kd) + j(\rho + 1)\sin(kd)} \quad \text{and} \quad \underline{b}_2 = \frac{\exp(-jkd) \underline{a}_1 - Z_c (0.5 \cdot \rho + 1) \underline{V}_p}{\cos(kd) + j(\rho + 1)\sin(kd)}$$

### 6.1. Case studies

#### 6.1.1. Null pressure on the rear surface of the MPP

If we want a surface impedance of value  $\underline{Z}(0) = R$ , we have to force the pressure on the internal face of the MPP to be zero.

**Proof:**

With  $\underline{p}_2(0) = \underline{a}_2 + \underline{b}_2 = 0$ , we have:  $\underline{V}_p = \left( \frac{2}{Z_c + R} \right) \underline{p}_{1+}(0) \cos(kd)$

We can see that:  $\underline{v}_2(0) = Z_c^{-1} (\underline{a}_2 - \underline{b}_2) = \frac{2 \underline{p}_{1+}(0)}{Z_c (1 + \rho)}$

$$\text{But: } \underline{v}_1(0) = Z_c^{-1}(\underline{a}_1 - \underline{b}_1) = \frac{\underline{p}_{1+}(0) - \underline{p}_{1-}(0)}{Z_c}$$

By continuity,  $\underline{v}_2(0) = \underline{v}_1(0)$  and so we find the well-known reflection coefficient of the surface impedance  $R$ :

$$\underline{p}_{1-}(0) = \frac{R - Z_c}{R + Z_c} \underline{p}_{1+}(0)$$

**Note:** this requires measuring the incident pressure  $\underline{p}_{1+}(0)$  (two microphones technique).

### 6.1.2. Piston velocity governed by the pressure in front of the MPP

If we impose a velocity  $\underline{V}_p = \underline{G}(f)(\underline{p}_{1+}(0) + \underline{p}_{1-}(0))R^{-1}$ , with  $(\underline{p}_{1+}(0) + \underline{p}_{1-}(0))$  *measured by one microphone*, we have:

$$R\underline{V}_p = \underline{G}(f)(\underline{a}_1 + \underline{b}_1) = \frac{2(\cos(kd) + j\rho \sin(kd))\underline{a}_1 - Z_c \underline{V}_p}{\cos(kd) + j(\rho + 1)\sin(kd)} \underline{G}(f)$$

$$\text{and } \underline{V}_p = \frac{2(\cos(kd) + j\rho \sin(kd))\underline{G}(f)}{R\cos(kd) + jR(\rho + 1)\sin(kd) + Z_c \underline{G}(f)} \underline{a}_1$$

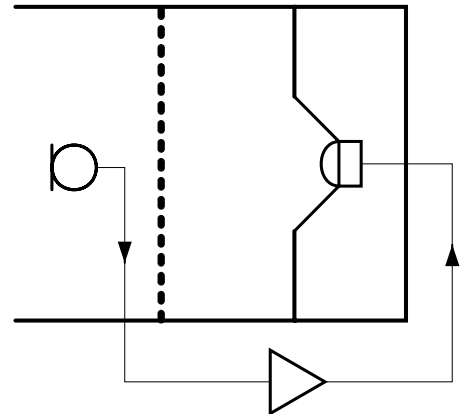
We can then find  $\underline{b}_1$  and  $\underline{r}(0) = \frac{\underline{b}_1}{\underline{a}_1}$ . The absorption is given by  $\alpha = 1 - |r|^2$ .

### 6.1.3. Electrodynamic loudspeaker used as velocity transducer

If we use an electrodynamic loudspeaker as a piston, the velocity is band-pass filtered. We know that the low-frequency model gives us a normalized transfer function of voltage to velocity [2], given by:

$$\underline{G}(f) = \frac{jQ_t^{-1}\left(\frac{f}{f_c}\right)}{1 + jQ_t^{-1}\left(\frac{f}{f_c}\right) - \left(\frac{f}{f_c}\right)^2}$$

where  $f_c$  is the resonance frequency and  $Q_t$  the total quality factor of the loudspeaker.





**Example:**

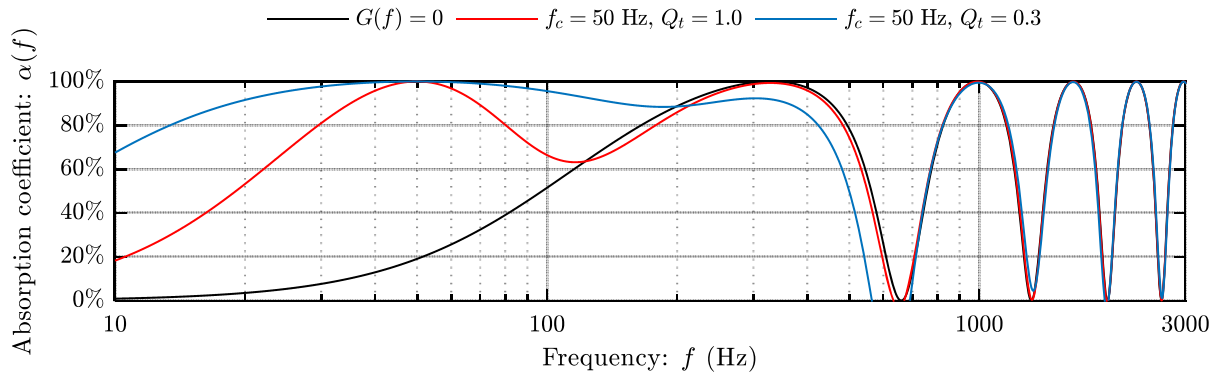


Figure 7. Example of absorption with  $Z_c = 400$  Pa·s/m and  $R = 400$  Pa·s/m.

We can see that it is possible to increase the absorption in low frequency. However, this requires a suited filter to guarantee stability.

## 7. 3D vs. 1D

In one dimension, the ideal impedance  $Z_{abs}$  that the absorber has to present is always the characteristic impedance of the medium  $Z_c$ .

If the absorber surface  $S_{abs}$  doesn't entirely cover the wall surface  $S$ , the ideal impedance is **lower** than  $Z_c$ , for reasons of volume flow conservation:

$$Z_{abs} = \frac{S_{abs}}{S} Z_c$$

However, one can easily understand that the absorber surface cannot be reduced too much: when the absorber surfaces are not evenly distributed on the wall, the wave isn't plane anymore and the last hypothesis isn't valid.

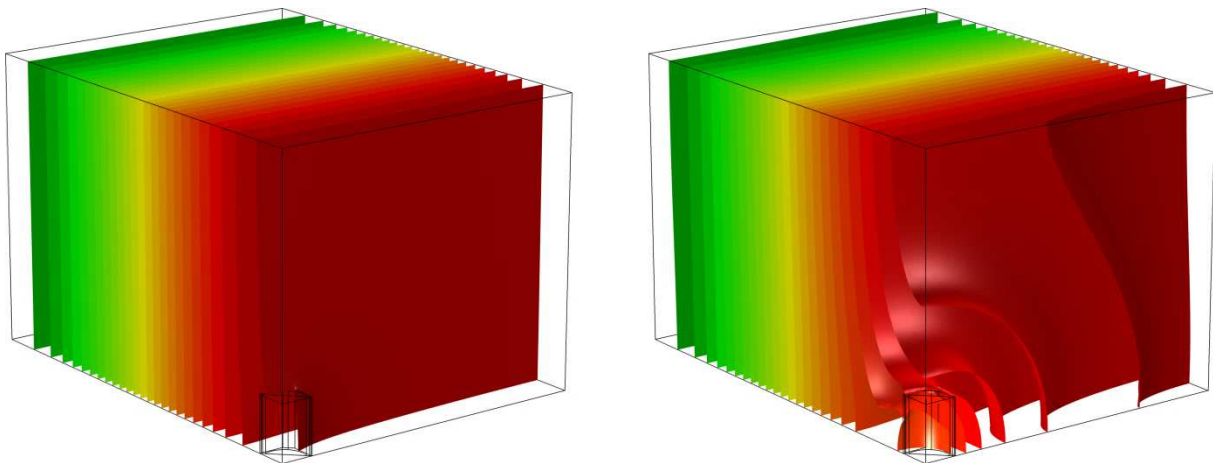


Figure 8. Structure of the 1<sup>st</sup> mode, with the absorber switched off (left) and on (right).

Figure 8 shows the structure of the 1<sup>st</sup> mode, with a 0.2 m<sup>2</sup> absorber, first switched off (left) then switched on (right). On the left, the wave is plane and propagates itself in the longest dimension of the room. On the right, the wave gets distorted by the absorber and tends to be spherical near the absorber. In this case, there's no analytical solution for the ideal absorber impedance and a FEM analysis is required [3].

All FEM figures were done with COMSOL Multiphysics® 4.3b.

We first conducted a parametric study on the volume of the room, the surface of the absorbers and their impedance  $Z_{abs}$ . For each case, the first 20 eigenfrequencies  $\underline{f} = f_\delta + j\delta$  were computed, then the natural frequencies  $f_0$  and the modal extinction times  $MT_{60}$  were extracted:

$$f_0 = |\underline{f}| \quad \text{and} \quad MT_{60} = \frac{3\ln(10)}{2\pi \text{Im}(\underline{f})}$$

Such a study is presented in Figure 9. For each mode, one can see that there is an optimal impedance  $Z_{abs}$  (black dots) leading to the shortest extinction time  $MT_{60}$ .

In this example, an absorber impedance  $Z_{abs}$  in the range 100–200 Pa·s/m would represent a good compromise for all modes.

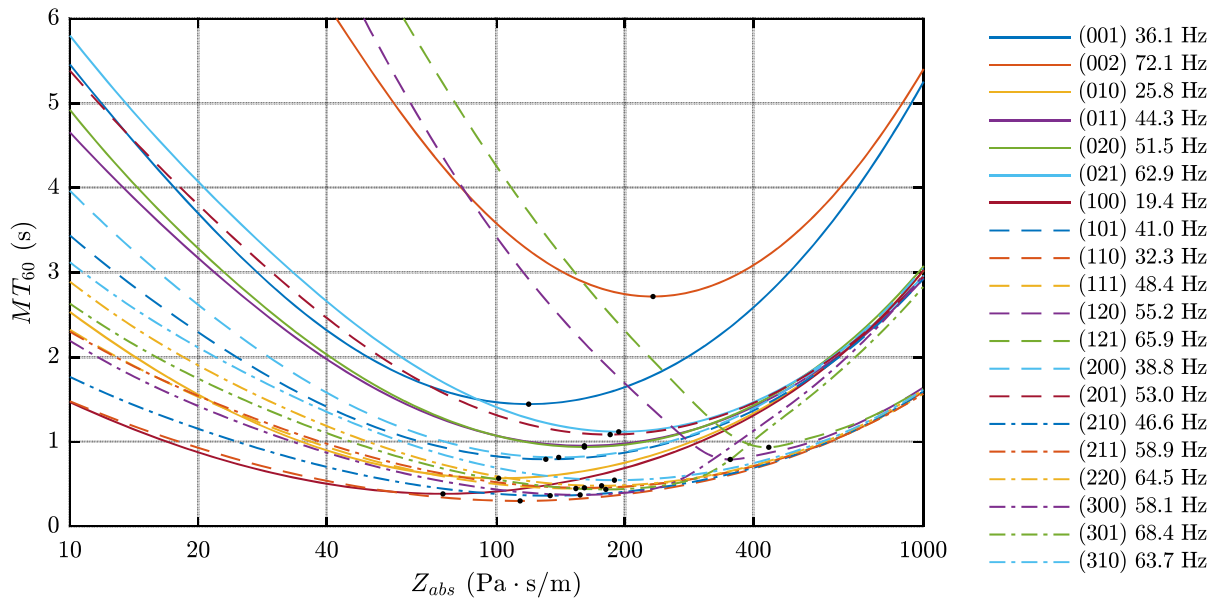


Figure 9. Modal extinction time as a function of  $Z_{abs}$  for the first 20 modes.

To confirm these studies, we then compared the instantaneous intensity travelling in the room as well as the sound pressure level, when an absorber of a given shape possesses successively the impedances 400 Pa·s/m and 100 Pa·s/m.

Figure 10 shows that sound pressure in the room is more homogenous when the absorber impedance  $Z_{abs} = 100$  Pa·s/m.

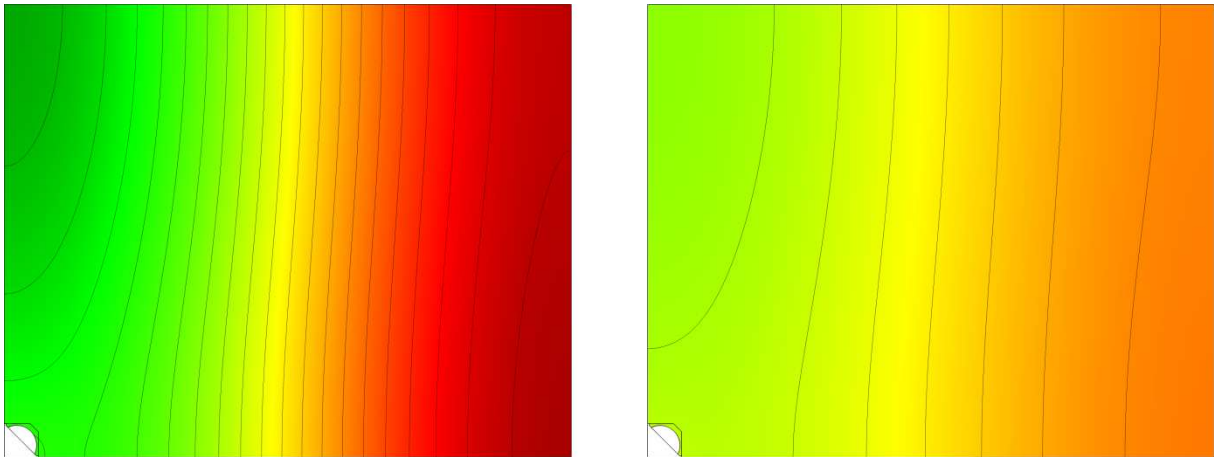


Figure 10. Sound pressure with isobars in the room, when  $Z_{abs} = 400$  Pa·s/m (left) and  $Z_{abs} = 100$  Pa·s/m (right).

If we examine the specific impedance in the room (Figure 11), we note that, in the first case (left picture,  $Z_{abs} = 400$  Pa·s/m), the surface having a specific impedance  $Z_s$  equal to the characteristic impedance of the medium  $Z_c$  is the absorber surface.

But, in the second case (right picture,  $Z_{abs} = 100$  Pa·s/m), there's a virtual surface of specific impedance  $Z_s = Z_c$  in front of the absorber (black circle). As this virtual surface is larger than the absorber surface, one can understand that the absorption would be greater.

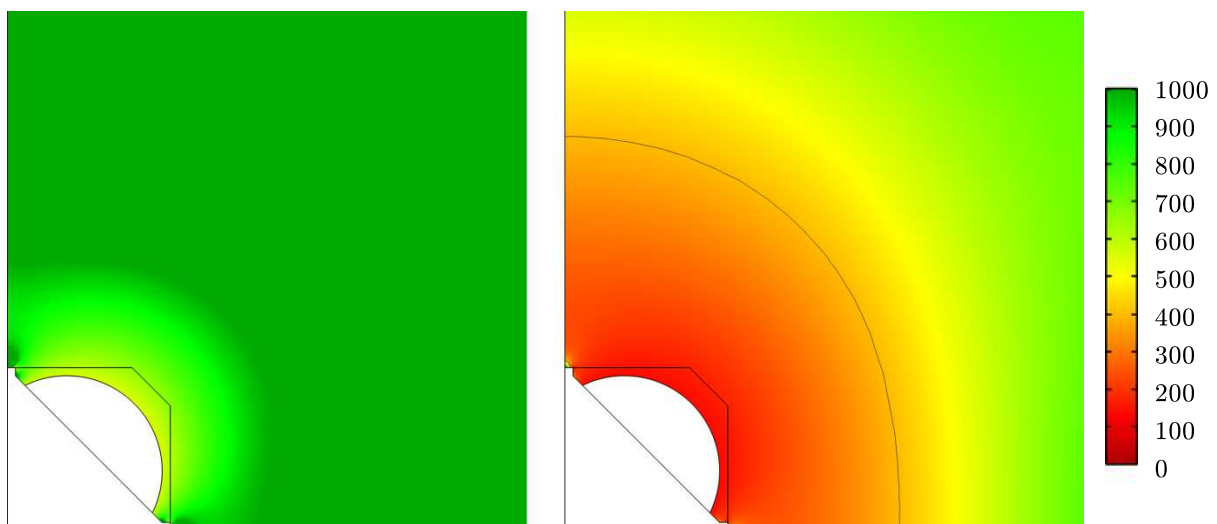


Figure 11. Specific impedance  $Z_s$ , when  $Z_{abs} = 400$  Pa·s/m (left) and  $Z_{abs} = 100$  Pa·s/m (right).  
The black circle on the right picture denotes the value  $Z_s = 400$  Pa·s/m =  $Z_c$ .

## 8. Implementation

Using the results of the previous studies, Relec SA built a patent-pending<sup>1</sup> absorber named *AVAA*. In order to cancel the internal pressure, the absorber uses a predictive setpoint (feedforward control). Considering the schematic given in Figure 12, the in-going volume flow rate  $\underline{q}$  has to match:

$$\underline{q} = \underline{v} \cdot A_f = \frac{P_{ext}}{R} \cdot A_f$$

where  $A_f$  is the MPP area and  $R$  the MPP flow resistance.

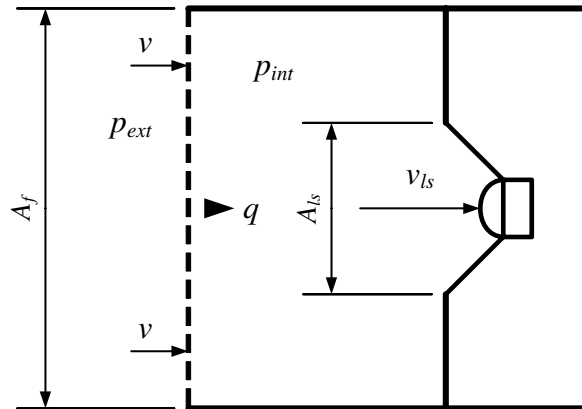


Figure 12. General principle of acoustic pressure cancellation behind a MPP using a velocity transducer.

This volume flow rate  $\underline{q}$  is realized with a velocity transducer. At low frequencies, the physical dimensions of the device are significantly smaller than the wavelength. Assuming volume flow rate continuity, the transducer velocity setpoint  $\underline{v}_{ls}$  is given by:

$$\underline{v}_{ls} = \frac{\underline{q}}{A_{ls}} = \frac{1}{R} \cdot \frac{A_f}{A_{ls}} \cdot \underline{p}_{ext}$$

where  $A_{ls}$  is the projected transducer membrane area. The absorption area is significantly increased by this method, as  $A_f$  can be easily ten times bigger than  $A_{ls}$ .

Figure 13 shows a schematic of the absorber, starting with the MPP (5). These panels are manufactured with precise and well-known characteristics and with flow resistance lower than  $Z_c$ . The current version of the absorber includes a 0.5 mm thick MPP having a porosity of 5% and an air-flow resistance of about 180 Pa·s/m (imaginary part kept below 25 Pa·s/m in the range 0–200 Hz). The MPP (5) forms the front side of a closed volume (4), of which the back side is a baffle (2) including one or more velocity transducers (1). The transducers are then mounted on a closed rear volume (3).

<sup>1</sup> UK patent application number GB1421213.8

The acoustic pressure in front of the MPP (5) is acquired by a microphone (8). The pressure signal is then converted to an appropriate voltage level by a preamplifier (9). A feedforward control (10) takes the preamplifier output signal and drives a power amplifier input (6), including a transfer function  $H_1$  given by:

$$H_1 = \frac{A_f}{R \cdot A_{ls}} \cdot \frac{1}{G_1}$$

where  $A_f$  is the MPP area (5),  $A_{ls}$  the projected transducer membrane area (1) and  $G_1$  the preamplifier (9) gain.

The feedforward control (10) also includes a band-pass filter to control the bandwidth of the system and guarantee its stability.

To end with, the power amplifier (6) uses a measurement (7) of the transducer (1) membrane velocity in a feedback loop in order that the membrane velocity matches the input signal of the amplifier.

The velocity transducer is based on the same idea as the impedance bridge shown in Figure 14, where the input voltage  $\underline{V}_{in}$  is the power amplifier input.

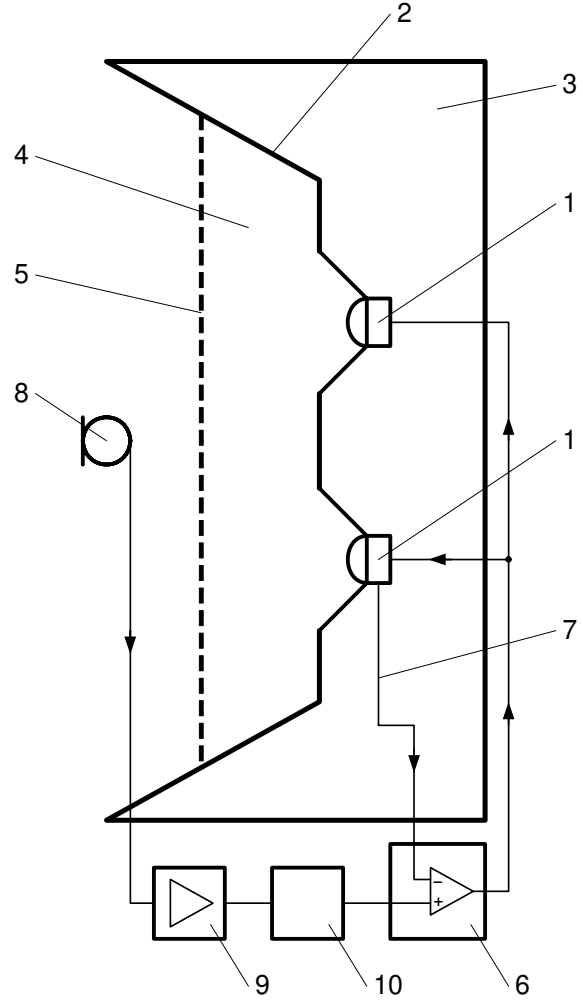


Figure 13. Schematic of the implementation.

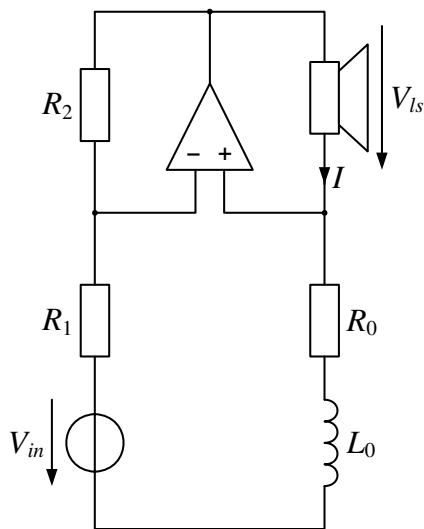


Figure 14. Voltage to acoustic velocity converter used in the power amplifier.

The voltage  $\underline{V}_{ls}$  is given by:

$$\underline{V}_{ls} = \underline{Z}_{ls} \cdot \underline{I} + Bl \cdot \underline{v}_{ls}$$

where  $\underline{Z}_{ls} = R_e + j\omega \cdot L_e$  is the electric impedance of the loudspeaker,  $\underline{I}$  the current through the loudspeaker coil,  $Bl$  the force factor and  $\underline{v}_{ls}$  the membrane velocity.

Resistor  $R_0$  is chosen small in order to save power. Resistors  $R_1$  and  $R_2$  are proportional to  $R_0$  and  $R_e$  respectively.

Inductor  $L_0$  is given by:  $L_0 = \frac{R_0}{R_e} L_e$

Hence the induced voltage in the loudspeaker coil  $Bl \cdot \underline{v}_{ls}$  is proportional to the input voltage  $\underline{V}_{in}$ . This leads to a membrane velocity given by:

$$\underline{v}_{ls} = \frac{1}{Bl} \cdot \frac{R_e}{R_0} \cdot \underline{V}_{in}$$

In practical applications, this bridge is realized without the inductor  $L_0$ , but rather with complex impedances  $\underline{Z}_1$  and  $\underline{Z}_2$  in place of resistors  $R_1$  and  $R_2$  respectively.

This is also true when a more accurate loudspeaker model is used for  $\underline{Z}_{ls}$ , e.g. to account for eddy currents, according to [4]. The accuracy of this model will determine the bandwidth of the system.

## 9. Validation

### 9.1. Measurement method

The extinction times can be measured using two different methods: time domain and frequency domain.

In *time domain*, each mode is measured one after the other. First, the mode is excited with a sine wave. Then the excitation is stopped and the sound pressure level decay in the room is recorder over time. The modal extinction time is defined as the time elapsed for a SPL decrease of 60 dB.

We rather choose to conduct measurement in the *frequency domain*.

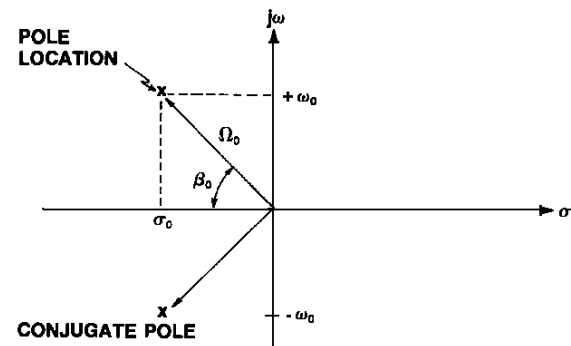
First, the room frequency response function (pressure in one corner relative to the pressure in the opposite corner or relative to the source volume flow) is acquired with pink noise excitation, FFT and averaging during several minutes.

Then the poles are extracted using the rational fraction polynomials method from Richardson and Formenti [5],[6]:

- The frequency response function (FRF) is expressed as rational fraction polynomials, with degrees  $m$  and  $n$  depending on the number of peaks:

$$\underline{H}(\omega) = \frac{\sum_{k=0}^m \underline{a}_k \underline{s}^k}{\sum_{k=0}^n \underline{b}_k \underline{s}^k} \Big|_{\underline{s}=j\omega}$$

- $\underline{a}_k$  and  $\underline{b}_k$  are obtained by linear regression;



$\sigma_0$  – DAMPING COEFFICIENT  
 $\omega_0$  – DAMPED NATURAL FREQUENCY  
 $\Omega_0$  – RESONANT (UNDAMPED) NATURAL FREQUENCY

- The FRF is then converted into a sum of simple fractions:

$$\underline{H}(\omega) = \sum_{k=1}^{n/2} \left( \frac{\underline{r}_k}{\underline{s} - \underline{p}_k} + \frac{\underline{r}_k^*}{\underline{s} - \underline{p}_k^*} \right) \bigg|_{\underline{s}=j\omega} \quad \text{with} \quad \underline{p}_k = -\sigma_k + j\omega_k$$

- Finally, for each pole, the natural frequency is given by:  $f_0 = \frac{|p|}{2\pi}$

The quality factor is given by:  $Q = \frac{-|p|}{2\text{Re}(p)}$

And the extinction time is obtained by:  $MT_{60} = \frac{3\ln(10) Q}{\pi f_0}$

This method was validated on first measurements, as illustrated on Figure 15. The results are similar when the modes aren't too close. Besides, it's also difficult to excite two close modes separately in time domain.

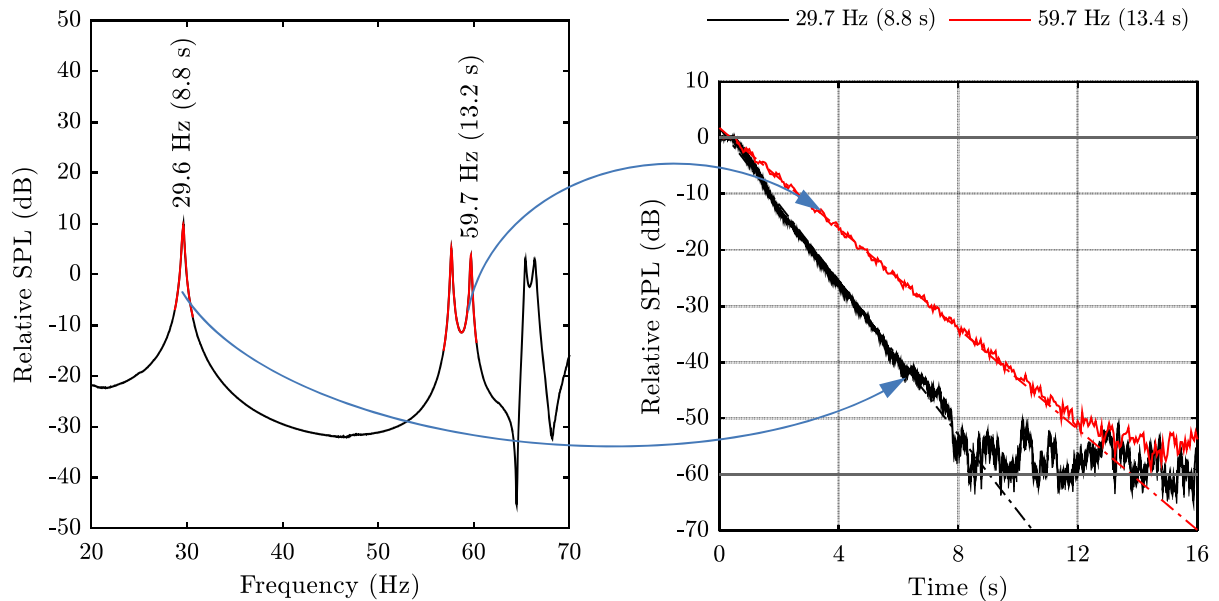


Figure 15.  $MT_{60}$  computation in frequency (left) and time domain (right).

Using Sabine's formula, one can then estimate the **equivalent absorbent area**  $A_{eq}$  from the extinction time with and without the absorber  $T_{on}$  and  $T_{off}$  :

$$A_{eq} = 0.161 V \left( \frac{1}{T_{on}} - \frac{1}{T_{off}} \right)$$

## 9.2. Results

Figure 16 shows an estimation of the impedance  $\underline{Z}_{abs}$  presented by the absorber, computed from a measurement of the velocity transducer FRF. We can define the bandwidth to be roughly 10–150 Hz.

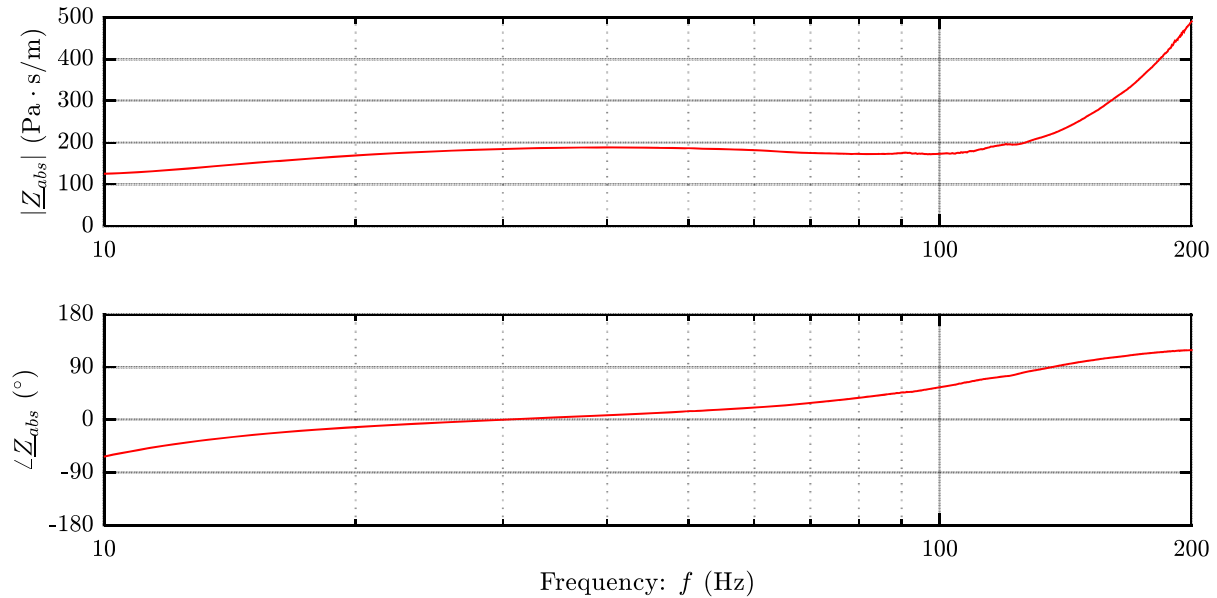


Figure 16. Absorber impedance  $\underline{Z}_{abs}$  vs. frequency.

The AVAA is being tested in many premises: studios, listening rooms, etc. In a small test room at hepia (approximately  $6.0 \times 2.6 \times 2.9$  m), the equivalent absorbent areas  $A_{eq}$  of the first modes are:

$f$ (Hz)	Mode	$T_{off}$ (s)	$T_{on}$ (s)	$A_{eq}$ (m <sup>2</sup> )
29.3	100	4.75	0.99	5.8
59.9	010	8.16	2.60	1.9

The comparison of the room frequency response functions with and without the absorber is given in Figure 17.

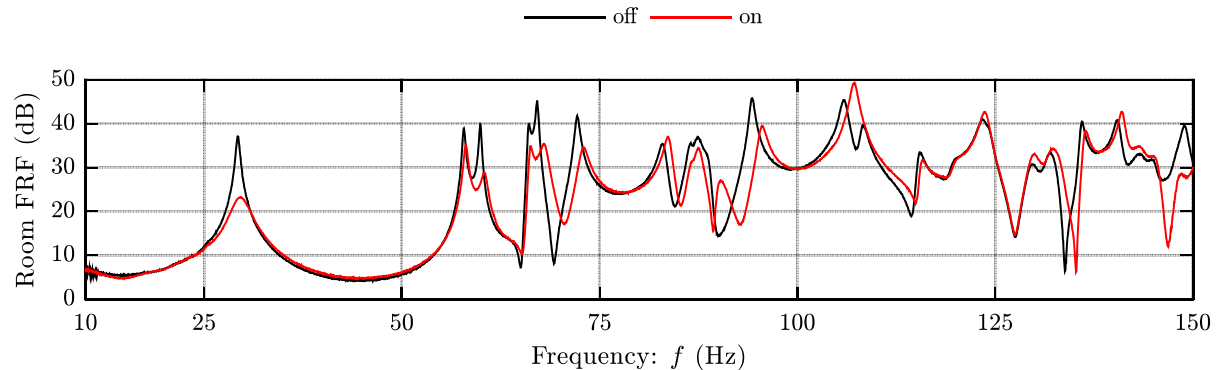


Figure 17. Comparison of the test room FRF with the absorber switched off and on.



Using the impedance estimation shown in Figure 16, the equivalent absorbent area  $A_{eq}$  of the AVAA was simulated on the first mode of rooms from 6 m<sup>2</sup> to 54 m<sup>2</sup>, with walls absorption coefficients from  $\alpha = 0.050$  to  $\alpha = 0.125$ , for one, two and four absorbers. The results are given in Figure 18.

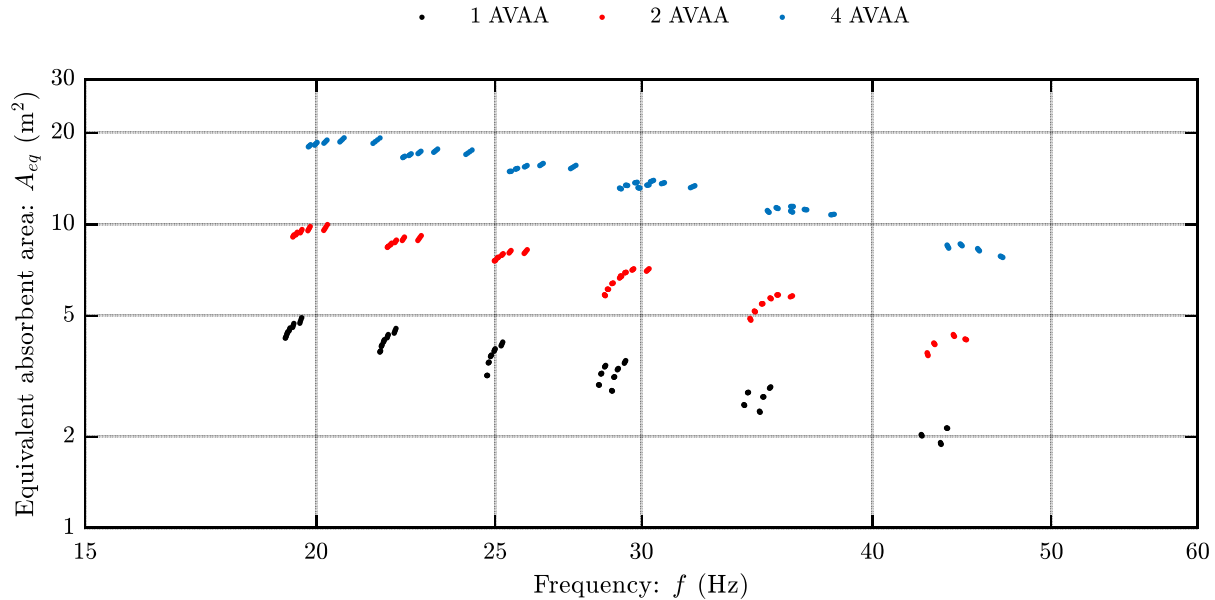


Figure 18. Simulated equivalent absorbent area  $A_{eq}$  on the first mode of several rooms.

## 10. Further development

In order to increase the precision of the internal pressure cancellation, it is possible to add a feedback control loop using the internal pressure  $\underline{p}_{int}$  (see Figure 12).

As the internal pressure setpoint is zero, the pressure  $\underline{p}_{int}$  is equivalent to an error signal that can be used directly: a positive internal pressure has to produce a positive transducer velocity, according the figure.

With this additional control loop, the velocity setpoint becomes:

$$\underline{v}_{ls} = \frac{1}{R} \cdot \frac{A_f}{A_{ls}} \cdot \underline{p}_{ext} + K \cdot \underline{p}_{int}$$

where the feedback gain  $K$  is chosen significantly larger than the feedforward gain  $A_f \cdot A_{ls}^{-1} \cdot R^{-1}$ .

## 11. References

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